

# Dark energy as a massive vector field

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## Abstract

We propose that the Universe is filled with a massive vector field, non-minimally coupled to gravitation. The field equations of the model are consistently derived and their application to cosmology is considered. The Friedmann equations acquire an extra dark-energy component, which is proportional to the mass of the vector particle. This leads to a late-time accelerated de Sitter type expansion. The free parameters of the model (gravitational coupling constants and initial value of the cosmological vector field) can be estimated by using the PPN solar system constraints. The mass of the cosmological massive vector particle, which may represent the main component of the Universe, is of the order of  $10^{-63}$  g.

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## I. INTRODUCTION

An increasing number of observational data, obtained in the past few years, strongly support a model according to which the Universe is spatially flat, mostly made of non-conventional matter – baryons being allowed only up to 4% of the total energy content – and accelerating. The physical models accounting for such a picture generally contain two basic ingredients: pressureless dark matter (DM), responsible for the growth of cosmological perturbations via gravitational instability, and negative pressure dark energy (DE), responsible for the accelerated expansion (for recent reviews on the dark energy problem see [1]).

The simplest model along these lines is  $\Lambda$ CDM, in which the role of DE is played by a cosmological constant  $\Lambda$ . It fits very well all the data related with the cosmological background and the perturbations in the linear regime [2]. The cosmological constant is attributed to the quantum zero-point energy of the particle physics vacuum, with a constant energy density  $\rho$ , pressure  $p$  and an equation of state  $w = p/\rho = -1$ . Existing observational data indicate the equation of state of dark energy very close to the cosmological constant value,  $w = -1 \pm 0.2$  at 95% confidence level, with at most a very mild evolution up to redshift  $z \sim 1$ .

An alternative model to the cosmological constant may be the quintessence [3], a dynamical scalar field which, in the simplest model, slowly rolls in a potential characterized by an extremely low mass. The spatially averaged equation of state for the quintessence field satisfies  $w > -1$ . In order to avoid fine-tuning on the initial conditions, the quintessence scalar field is usually taken to be extremely light, with a Compton wavelength corresponding to the present value of the Hubble radius. As a consequence, the scalar field is homogeneous on all observable scales, much like a cosmological constant. However, quintessence models suffer from an important theoretical problem, namely, the fact that radiative corrections induced by the couplings with the matter fields would generically induce huge corrections to the trace level mass, thus spoiling the required lightness. Hence to keep the scalar field light in these models a fine-tuning on the radiative corrections is generally required, besides the one necessary to keep the cosmological constant small.

A wide variety of other dark energy models has also been proposed, including K-essence [4], Chaplygin gas [5], modifications of gravity [6], Born-Infeld scalars (rolling

tachyon) [7], massive scalars [8] etc. The common feature of these models is that they operate through an undetermined field potential which in principle can incorporate any a priori associated cosmological evolution, thus lacking predictive power at the fundamental level [7]. There is a tremendous degeneracy in these models and generally they are judged by their physical implications and by the generic features which arise in them. Therefore a consistent physical picture of the dark energy, which could explain the size of its energy density  $\rho_{\text{DE}} \approx 10^{-12} \text{ eV}^4$ , and suggests how the underlying physics may be probed, is still missing.

Non-gravitational interactions are known to be mediated by vector fields. Therefore the possibility that a vector field, which, for example, may be a partner of quintessence, could be at the origin of the present stage of the cosmic acceleration cannot be neglected. Theoretical proposals in which a minimally coupled vector field is responsible for the present dynamics of the Universe have been considered in [9].

It is the purpose of the present paper to propose a model of the dark energy in terms of a massive, Proca type vector field, with a non-minimal coupling to the gravitational field. The model contains three independent parameters  $\omega$ ,  $\eta$  and  $\mu_\Lambda^2$ , respectively, with  $\mu_\Lambda^2$  representing the mass of the massive cosmological vector particle. The scalars  $\omega$  and  $\eta$  describe the non-minimal coupling of the vector field to the Ricci scalar and to the Ricci tensor, respectively. The gravitational field equations can be consistently derived from a variational principle. A similar vector-tensor theory, without the mass term, was also proposed in the early 1970's [10].

In the cosmological case, corresponding to a flat homogeneous and isotropic Universe, the Friedmann equations acquire an extra dark-energy component, which is proportional to the mass of the vector particle. This term, playing the role of the cosmological constant, leads to the late accelerated expansion of the Universe. The free parameters of the model (gravitational coupling constants and initial value of the cosmological vector field) can be estimated by using the PPN solar system constraints. The mass of the cosmological massive vector particle, which may represent the main component of the Universe, is of the order of  $10^{-63} \text{ g}$ .

The present paper is organized as follows. The field equations of the massive vector-tensor theory are derived in Section II. In Section III we consider the cosmological applications of the model. The PPN constraints on the model parameters are discussed in Section IV. We

discuss and conclude our results in Section V.

Throughout this paper we use the Landau-Lifshitz conventions [11] for the metric signature  $(+, -, -, -)$  and for the field equations, and a system of units with  $c = \hbar = 1$ .

## II. FIELD EQUATIONS OF THE MASSIVE VECTOR-TENSOR THEORY

We assume that the Universe is filled with a massive cosmological vector field, with mass  $\mu_\Lambda$ , which is characterized by a four-potential  $\Lambda^\mu(x^\nu)$ ,  $\mu, \nu = 0, 1, 2, 3$  and which couples non-minimally to gravity. In analogy with electrodynamics we introduce the field tensor

$$C_{\mu\nu} = \nabla_\mu \Lambda_\nu - \nabla_\nu \Lambda_\mu. \quad (1)$$

The interaction of the gravitational and of the vector fields is described by a Lagrangian which is required to satisfy the following conditions: a) the Lagrangian density is a four-scalar b) the free-field energies are positive-definite for both the metric and the vector field c) the resulting theory is metric and d) the field equations contain no higher than second derivatives of the fields [10]. The action for such a theory can be written as

$$S = - \int \left[ R + C_{\mu\nu} C^{\mu\nu} + \frac{1}{2} \mu_\Lambda^2 \Lambda_\mu \Lambda^\mu + \omega \Lambda_\mu \Lambda^\mu R + \eta \Lambda^\mu \Lambda^\nu R_{\mu\nu} + 16\pi G_0 L_m \right] \sqrt{-g} d\Omega, \quad (2)$$

where  $R_{\mu\nu}$  and  $R$  are the Ricci tensor and the Ricci scalar, respectively,  $G_0$  is the gravitational constant and  $L_m$  is the matter Lagrangian. In Eq. (2)  $\omega$  and  $\eta$  are dimensionless coupling parameters. The four-dimensional volume element is  $d\Omega = dx^0 dx^1 dx^2 dx^3$ . In the following we denote  $\phi = \Lambda_\mu \Lambda^\mu$ , which is an invariant scalar.

The variation of the action with respect to the metric tensor  $g_{\mu\nu}$  gives the field equations

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \omega \left[ \phi \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + \Lambda_\mu \Lambda_\nu R + g_{\mu\nu} \nabla_\lambda \nabla^\lambda \phi - \nabla_\nu \nabla_\mu \phi \right] + \\ \eta \Lambda^\alpha \Lambda^\beta \left( g_{\mu\alpha} R_{\nu\beta} + g_{\nu\beta} R_{\mu\alpha} - \frac{1}{2} g_{\mu\nu} R_{\alpha\beta} \right) + \\ \frac{\eta}{2} \left[ g_{\mu\nu} \nabla_\alpha \nabla_\beta (\Lambda^\alpha \Lambda^\beta) + \nabla_\sigma \nabla^\sigma (\Lambda_\mu \Lambda_\nu) - \nabla_\sigma \nabla_\nu (\Lambda_\mu \Lambda^\sigma) - \nabla_\sigma \nabla_\mu (\Lambda_\nu \Lambda^\sigma) \right] + \\ 2C_{\mu\sigma} C_\nu^\sigma - \frac{1}{2} g_{\mu\nu} C_{\alpha\beta} C^{\alpha\beta} + \frac{1}{2} \mu_\Lambda^2 \Lambda_\mu \Lambda_\nu - \frac{1}{4} \mu_\Lambda^2 \phi g_{\mu\nu} = 8\pi G_0 T_{\mu\nu}, \end{aligned} \quad (3)$$

where  $T_{\mu\nu}$  is the energy-momentum tensor of the matter, defined in terms of the matter action  $S_m = 16\pi G_0 \int L_m \sqrt{-g} d\Omega$  as  $\delta S_m = 8\pi G_0 \int T_{\mu\nu} \delta g^{\mu\nu} \sqrt{-g} d\Omega$ .

The variation of the action with respect to  $\Lambda_\mu$  gives the generalized Maxwell equation

$$4\nabla_\nu C^{\mu\nu} + \omega R \Lambda^\mu + \eta \Lambda^\nu R_\nu^\mu + \frac{1}{2} \mu_\Lambda^2 \Lambda^\mu = 0. \quad (4)$$

By contracting the field equations we find

$$-R + \left(3\omega + \frac{\eta}{2}\right) \nabla_\lambda \nabla^\lambda \phi + \eta \nabla_\alpha \nabla_\beta (\Lambda^\alpha \Lambda^\beta) = 8\pi G T, \quad (5)$$

where  $T = T_\mu^\mu$ . Taking the covariant derivative of Eq. (4) we obtain the conservation law for the four-potential of the massive cosmological field as

$$\left(\frac{1}{2} \mu_\Lambda^2 + \omega R\right) \nabla_\mu \Lambda^\mu + \left(\omega + \frac{\eta}{2}\right) (\nabla_\mu R) \Lambda^\mu + \eta (\nabla_\mu \Lambda^\nu) R_\nu^\mu = 0. \quad (6)$$

From Eq. (4) it follows that the four-potential of the cosmological vector field satisfies the following wave equation:

$$\nabla_\sigma \nabla^\sigma \Lambda^\mu - \nabla^\mu (\nabla^\nu \Lambda_\nu) - \left[ \left(1 + \frac{\eta}{4}\right) R_\nu^\mu + \frac{1}{4} \left(\omega R + \frac{1}{2} \mu_\Lambda^2\right) \delta_\nu^\mu \right] \Lambda^\nu = 0. \quad (7)$$

Due to its antisymmetry, the massive vector field tensor automatically satisfies the equations

$$\nabla_\sigma C_{\mu\nu} + \nabla_\mu C_{\nu\sigma} + \nabla_\nu C_{\sigma\mu} = 0. \quad (8)$$

The matter energy momentum tensor  $T_{\mu\nu}$  satisfies the conservation law  $\nabla^\mu T_{\mu\nu} = 0$ , which can be verified by taking the covariant divergence of Eq. (3).

### III. COSMOLOGICAL APPLICATIONS

To investigate the cosmological implications of the massive vector-metric theory we adopt the flat Robertson-Walker metric for a homogeneous and isotropic Universe, given by

$$ds^2 = dt^2 - a^2(t) (dx^2 + dy^2 + dz^2), \quad (9)$$

where  $a(t)$  is the scale factor. The observed isotropy and homogeneity of the Universe requires that the massive vector field is a function of the cosmological time only. Hence we assume that the potential  $\Lambda^\mu$  has only one non-zero component,  $\Lambda^\mu = (\Lambda^0(t), 0, 0, 0)$ . The function  $\phi$  is given by  $\phi(t) = \Lambda_0(t) \Lambda^0(t)$ .

The non-zero components of the Ricci tensor and the Ricci scalar are given by  $R_{00} = -3\ddot{a}/a$ ,  $R_{\alpha\alpha} = (a\ddot{a} + 2\dot{a}^2) \delta_{\alpha\alpha}$ ,  $\alpha = 1, 2, 3$  and  $R = -6(\ddot{a}/a + \dot{a}^2/a^2)$ , respectively. Moreover,

we have  $\nabla_\alpha \nabla_\beta (\Lambda^\alpha \Lambda^\beta) = \ddot{\phi} + 3\phi\ddot{a}/a + 6\phi\dot{a}^2/a^2 + 6\dot{\phi}\dot{a}/a$ ,  $\nabla_\sigma \nabla^\sigma (\Lambda_0 \Lambda_0) = \ddot{\phi} + 3\dot{\phi}\dot{a}/a - 6\phi\dot{a}^2/a^2$  and  $\nabla_\sigma \nabla_0 (\Lambda_0 \Lambda^\sigma) = \ddot{\phi} + 3\dot{\phi}\dot{a}/a - 3\phi\dot{a}^2/a^2$ .

We assume that the matter content of the Universe consists of the massive cosmological vector field  $C_{\mu\nu}$  and ordinary matter in form of pressureless dust with density  $\rho$ . The conservation of the energy-momentum tensor then gives  $\rho = \rho_0/a^3$ , where  $\rho_0$  is a constant of integration.

Hence, the gravitational field equations and the equation of motion of the vector field become

$$[1 - (\omega - \eta)\phi] \frac{\dot{a}^2}{a^2} - (2\omega + \eta)\phi \frac{\ddot{a}}{a} + \left(\omega + \frac{\eta}{2}\right) \dot{\phi} \frac{\dot{a}}{a} + \frac{\mu_\Lambda^2}{12}\phi = \frac{8\pi G_0 \rho_0}{3} \frac{1}{a^3}, \quad (10)$$

$$[2 + (2\omega + 3\eta)\phi] \frac{\ddot{a}}{a} + [1 + (\omega + 3\eta)\phi] \frac{\dot{a}^2}{a^2} + \left(\omega + \frac{\eta}{2}\right) \ddot{\phi} + 3(\omega + \eta) \dot{\phi} \frac{\dot{a}}{a} - \frac{\mu_\Lambda^2}{4}\phi = 0, \quad (11)$$

$$(2\omega + \eta) \frac{\ddot{a}}{a} + 2\omega \frac{\dot{a}^2}{a^2} = \frac{\mu_\Lambda^2}{6}. \quad (12)$$

Eliminating with the use of Eq. (12) the second derivative of the scale factor from Eq. (10) gives

$$[1 + (\omega + \eta)\phi] \frac{\dot{a}^2}{a^2} + \left(\omega + \frac{\eta}{2}\right) \dot{\phi} \frac{\dot{a}}{a} - \frac{\mu_\Lambda^2}{12}\phi = \frac{8\pi G_0 \rho_0}{3} \frac{1}{a^3}. \quad (13)$$

The general solution of Eq. (12) can be represented in an integral form as

$$t - t_0 = \int \frac{da}{\sqrt{\frac{\mu_\Lambda^2}{6(4\omega + \eta)} a^2 + a_0^2 a^{-\frac{4\omega}{2\omega + \eta}}}}, \quad (14)$$

where  $t_0$  and  $a_0$  are arbitrary constants of integration.

In the limit of large  $a$  and for  $4\omega/(2\omega + \eta) > 0$  the scale factor is given by

$$a = \exp[H(t - t_0)], \quad H = \text{constant}, \quad (15)$$

corresponding to a de Sitter type exponentially accelerating phase for the expansion of the Universe. The constant  $H$  is expressed in terms of the mass of the cosmological vector field as

$$H^2 = \frac{\mu_\Lambda^2}{6(4\omega + \eta)} = \frac{\Lambda}{3}, \quad (16)$$

where  $\Lambda$  is the cosmological constant, which is generated due to the presence of the massive cosmological vector particle, and whose numerical value can be determined from observations.

For arbitrary times the exact form of the scale factor is given by

$$a(t) = a_0 \sinh^n [\beta(t - t_0)], \quad (17)$$

where we have denoted

$$n = \frac{2\omega + \eta}{4\omega + \eta}, \quad (18)$$

and

$$\beta = \frac{(4\omega + \eta) H}{(2\omega + \eta)} = \frac{H}{n}, \quad (19)$$

respectively.

With the use of Eq. (17) it follows that the massive cosmological vector field satisfies the evolution equation

$$\begin{aligned} \dot{\phi} = \beta \left[ \tanh \beta (t - t_0) - \frac{2(\omega + \eta)}{4\omega + \eta} \coth \beta (t - t_0) \right] \phi - \frac{2H}{2\omega + \eta} \coth \beta (t - t_0) \\ + \frac{16\pi G_0 \rho_0}{3(2\omega + \eta) a_0^3 H} \frac{\sinh^{1-3n} \beta (t - t_0)}{\cosh \beta (t - t_0)}, \end{aligned} \quad (20)$$

with the general solution given by

$$\begin{aligned} \phi(t) = \frac{\cosh [\beta (t - t_0)]}{\sinh^{\frac{2(\omega + \eta)}{4\omega + \eta}} [\beta (t - t_0)]} \left\{ B + \frac{16\pi G_0 \rho_0}{3(4\omega + \eta) a_0^3 H^2} \tanh [\beta (t - t_0)] \right. \\ \left. - \frac{2}{4\omega + \eta} F [\beta (t - t_0)] \right\}, \end{aligned} \quad (21)$$

where  $B$  is an arbitrary constant of integration, and

$$F(x) = \int \frac{dx}{\sinh^m(x)}, \quad (22)$$

where

$$m = \frac{2\omega - \eta}{4\omega + \eta}. \quad (23)$$

During the pure de Sitter phase, with the effect of the ordinary matter neglected, the equation describing the dynamics of the time varying cosmological vector field  $\phi$  is

$$\dot{\phi} = \frac{2\omega - \eta}{2\omega + \eta} H \phi - \frac{2}{2\omega + \eta} H, \quad (24)$$

with the general solution

$$\phi(t) = \phi_0 \exp \left( \frac{2\omega - \eta}{2\omega + \eta} H t \right) + \frac{2}{2\omega - \eta} \left[ 1 - \exp \left( \frac{2\omega - \eta}{2\omega + \eta} H t \right) \right], \quad (25)$$

where  $\phi_0$  is the initial value of the field at the initial time  $t = t_0 = 0$ ,  $\phi(0) = \phi_0$ .

In the case of a constant field  $\phi = \phi_0 = \text{constant}$ ,  $H$  and  $\phi$  are related by

$$H^2 = \frac{\mu_\Lambda^2}{12} \frac{\phi_0}{1 + (\omega + \eta) \phi_0}, \quad (26)$$

from which we find the constant massive cosmological vector field as

$$\phi_0 = \frac{4\Lambda/\mu_\Lambda^2}{1 - 4(\omega + \eta)\Lambda/\mu_\Lambda^2} = \frac{2}{2\omega - \eta}. \quad (27)$$

#### IV. PPN CONSTRAINTS ON THE MODEL PARAMETERS

The numerical values of the coupling coefficients  $\omega$  and  $\eta$  and the initial value  $\phi_0$  of the cosmological vector field can be constrained by using solar system observations. Vector tensor models generate observable effects in light deflection and retardation experiments, planetary perihelion advance, orbiting gyroscope precession, non-secular terms in planetary and satellite orbits, geophysical phenomena etc. These effects can be described in terms of the dimensionless parameters  $\alpha$ ,  $\beta$  and  $\gamma$ , which parameterize deviations with respect to standard general relativity. Actually, the parameter  $\alpha$  can be settled to the unity due to the mass definition of the system itself [12].

The quantity  $\gamma - 1$  measures the degree to which gravity is not a purely geometric effect, and it is affected by other fields. Measurements of the frequency shift of the radio photons to and from the Cassini spacecraft as they passed near the Sun give the result  $\gamma = 1 + (2.1 \pm 2.3) \times 10^{-5}$  [13]. The value of the parameter  $4\beta - \gamma - 3$  can be constrained from Lunar Laser Ranging, with the observational result  $4\beta - \gamma - 3 = -(0.7 \pm 1) \times 10^{-3}$  [14]. For the cosmological time variation of the effective gravitational constant we adopt the value  $\dot{G}/G = 10^{-14} \text{ yr}^{-1}$  [15]. As for the present value of the Hubble constant we take  $H = 70 \text{ km/s/Mpc}$ .

Therefore, the three free parameters of our model  $(\phi_0, \omega, \eta)$  can be obtained from the following non-linear system of algebraic equations:

$$\gamma - 1 = \frac{2\omega(1 + \omega - \eta/2)\bar{\phi}}{1 - \omega(1 + 4\omega)\bar{\phi}} = 2.1 \times 10^{-5}, \quad (28)$$

$$4\beta - \gamma - 3 = \frac{(\gamma - 1)(1 - \omega\bar{\phi})}{\omega\bar{\phi}} \left\{ 1 + \frac{\gamma(\gamma - 2)}{\frac{1}{2}(\gamma + 1) - \frac{\eta(\gamma - 1)}{4\omega} + \left[ \frac{3\omega(\gamma - 1)}{2} + \frac{\eta(\gamma - 3)}{4} \right] \bar{\phi}} \right\} = -0.7 \times 10^{-3}, \quad (29)$$

$$\frac{1}{H} \frac{\dot{G}}{G} = \frac{\frac{3\omega(\gamma - 1)}{2} + \frac{\eta(\gamma - 3)}{4}}{2\omega + \eta} \frac{[(2\omega - \eta)\phi_0 - 2] \exp\left(\frac{2\omega - \eta}{2\omega + \eta}\right)}{\left\{ \frac{1}{2}(\gamma + 1) - \frac{\eta(\gamma - 1)}{4\omega} + \left[ \frac{3\omega(\gamma - 1)}{2} + \frac{\eta(\gamma - 3)}{4} \right] \bar{\phi} \right\}} = 1.40 \times 10^{-4}, \quad (30)$$

where we denoted by  $\bar{\phi}$  the present day value of the cosmological vector field,

$$\bar{\phi} = \phi \left( \frac{1}{H} \right) = \left( \phi_0 - \frac{2}{2\omega - \eta} \right) \exp\left( \frac{2\omega - \eta}{2\omega + \eta} \right) + \frac{2}{2\omega - \eta}. \quad (31)$$



These three equations are highly non-linear, and approximate solutions can only be obtained by the means of numerical methods.

To constrain the numerical values for the three free parameter, we firstly consider the  $(\omega, \phi_0)$ -plane for given values of the parameter  $\eta$  in Figures 1 and 2.

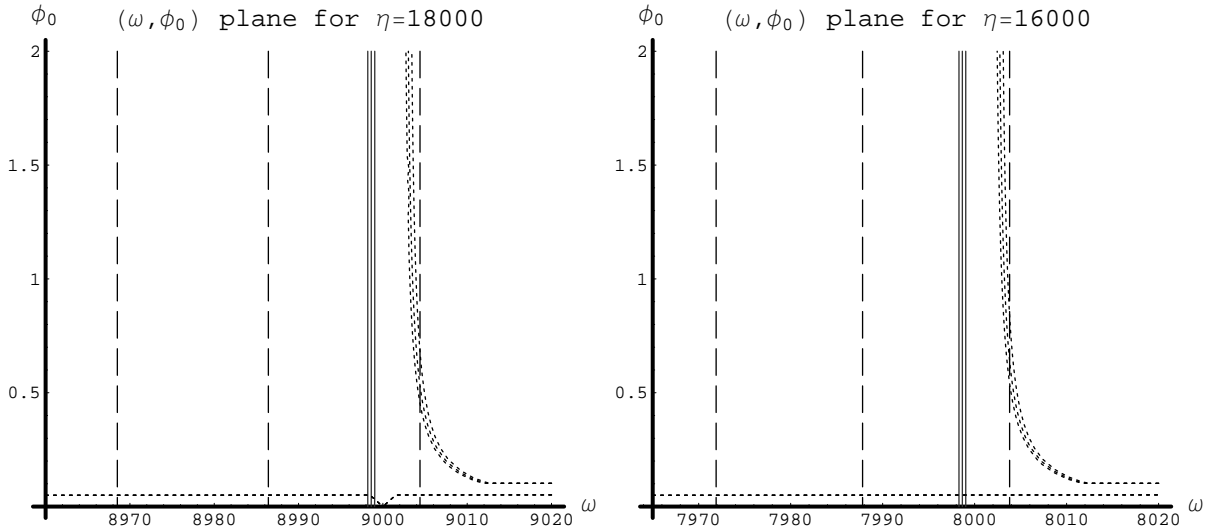


FIG. 1: These figures represent the  $(\omega, \phi_0)$ -planes for  $\eta = 18000$  and  $\eta = 16000$ . The three straight lines in the middle represent the allowed range of the parameter  $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ , as given by Eq. (28). The dashed lines represent the allowed range of  $4\beta - \gamma - 3$  via Eq. (29). The right dashed lines correspond to the value  $4\beta - \gamma - 3 = -(0.7 - 1) \times 10^{-3} = +0.3 \times 10^{-3}$  and implies that the model strongly favors positive values. Consequently, a further decrease on the error bars of  $4\beta - \gamma - 3$  can either rule out or further strengthen the present model. Lastly, the dotted lines describe the allowed parameter range for the ‘cosmological’ equation (30), where we used  $H = 70 \pm 10 \text{ km/s/Mpc}$ . It is evident that there is no parameter range where all three equations are satisfied to given accuracy, in fact one has to search for parameters that minimize the error.

The  $(\eta, \phi_0)$ -planes share features very similar to those of the  $(\omega, \phi_0)$ -planes, therefore it is not particularly insightful to also consider these. The reason behind this is that the numerical fitting of the three parameters works best if  $2\omega - \eta$  is of the order  $O(10^{-5})$ . The smallness of this difference has its roots in the Newtonian limit of the theory, which in case of a massless vector field reduces to  $2\omega = \eta$ . Hence, this small deviation is due to the massive vector field. However, let us consider the  $(\omega, \eta)$ -planes for given  $\phi_0$  in Figure 3.

For example, one possible set of numerical values for the free parameters of the model is

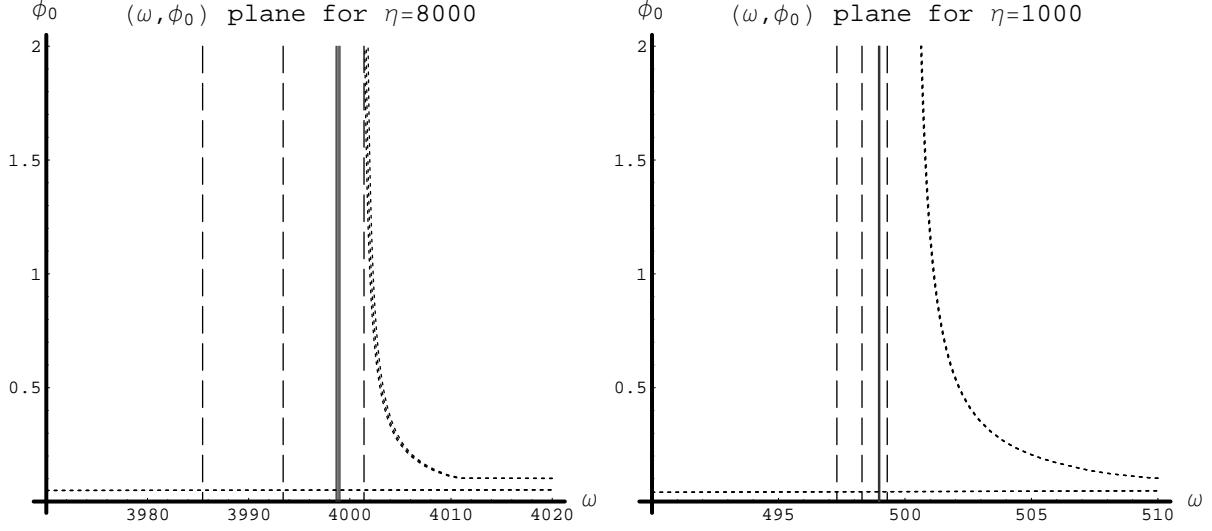


FIG. 2: Using the same coding as in the previous figure, it should be noted that the possibility of fitting the three parameters becomes more difficult. Already for  $\eta = 8000$  the ‘cosmological’ range of parameters (dotted) lies outside the allowed range of parameter as implied by Eq. (29) (dashed). Hence, it is even more difficult to fit the three parameters simultaneously. A further increase of  $\eta$  larger than 18000 has the same effect, the situation does not improve.

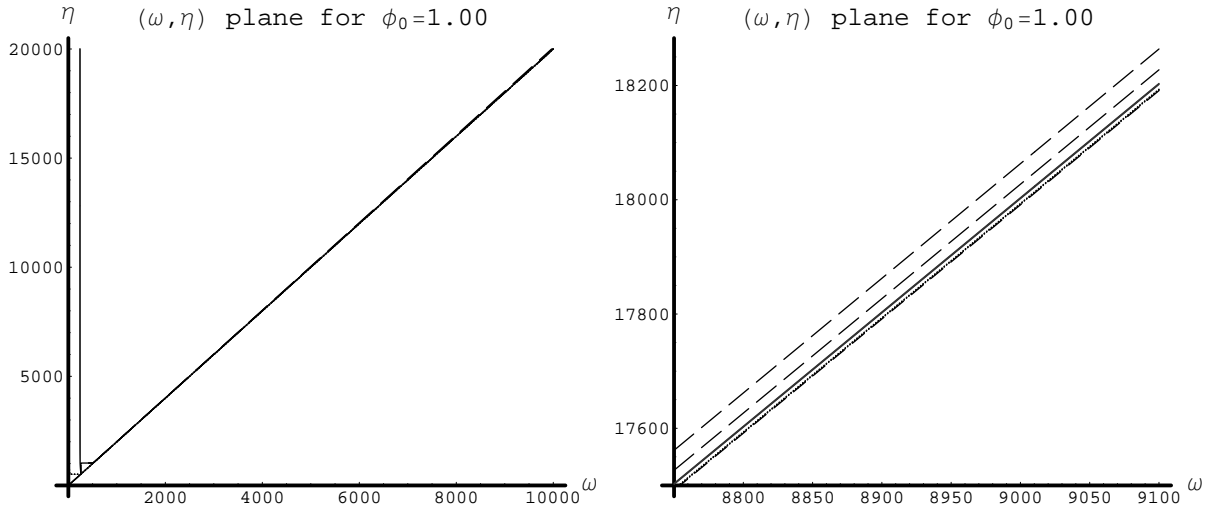


FIG. 3: As in the above figures, straight lines show the  $\gamma - 1$  range, dashed lines the  $4\beta - \gamma - 3$  range, and dotted lines the cosmological constraint. On the left-hand side, where  $0 < \omega < 10000$ , the different lines cannot be distinguished. The right-hand side ( $8750 < \omega < 9100$ ) again indicates that one can find some approximate solution for the three parameters. An increase of the parameter  $\phi_0$  does not change the qualitative picture.

given by  $\phi_0 = 1.097999982$ ,  $\omega = 9000.000069565$  and  $\eta = 18000.0001381$ , respectively. These values satisfy the general constraints on the coupling constants  $2\omega - \eta > 0$  and  $4\omega + \eta > 0$ .

The first condition is implied by the positivity of  $\phi_0$ , see Eq. (27), since  $\phi$  is the square of the massive vector field. The second condition follows from Eq. (16), i.e. positivity of the vector field mass and positivity of the Hubble constant. The above values represent an approximate solution of the system of constraint equations of the order of  $O(10^{-5})$ ,  $O(10^{-4})$  and  $O(10^{-4})$ . It is interesting to note that the system of equation is quite sensitive towards a change of the ‘cosmological’ equation (30).

If, for the moment, we assume that the gravitational constant  $G$  changes slower than given by the upper bound  $\dot{G}/G = 10^{-14} \text{ yr}^{-1}$ , for example an order of magnitude slower, then also the fit of the ‘cosmological’ equation improves by an order of magnitude. Hence, we can conclude that the analyzed theoretical model makes mild predictions regarding observation:

The gravitational constant should vary slower over time and the parameter  $4\beta - \gamma - 3$ , constrained by Lunar Laser Ranging, should lie in the positive region. On the other hand, it should be emphasized that our model would not be in good agreement with  $\dot{G} = 0$ , a constant effective gravitational “constant”.

## V. DISCUSSIONS AND FINAL REMARKS

From Eq. (16) we obtain for the mass of the cosmological vector particle the expression

$$\mu_\Lambda = \sqrt{6(4\omega + \eta)} H \approx 1.67 \times \sqrt{6(4\omega + \eta)} \times 10^{-63} \text{ g}. \quad (32)$$

The upper limit for the mass of the ordinary photon obtained by using a rotating torsion balance method is  $1.2 \times 10^{-51} \text{ g}$  [16]. It should be noted that the proposed massive vector field interacts only with gravity and has no standard matter interactions. The existence of a minimal mass in nature in the presence of a cosmological constant has been discussed for example in [17].

The mass of the cosmological massive vector particle may have been generated during inflation. Inflation allows the emergence of fields coherent over large distances from quantum fluctuations and prevents dissipative effects, due to the absence of charged particles. The conformal invariance of the U(1) gauge theory for electromagnetism prevents the gravitational field from producing photons. However, in the present model, due to the breaking

of the conformal invariance, via the coupling to gravity, the vector particle interacting with gravity can acquire an effective mass. Therefore, the effective mass for the massive vector particle composing the cosmological gas filling the Universe, and representing its main matter component, may have originated during inflation [18]. The mass of the cosmological vector particle can also be acquired from the spontaneous breaking of Lorentz symmetry in the context of field theories arising from string field theory, or due to the introduction of a fundamental minimal length in a trans-Planckian physics scenario [19].

The vector field dominated Universe enters in a pure exponential de Sitter phase when the condition  $t > t_{acc} = 1/\beta = (2\omega + \eta)t_H/(4\omega + \eta)$  is satisfied. With the use of the values of the parameters  $\omega$  and  $\eta$  obtained from the PPN solar system constraints we obtain  $t_{acc} \approx 2t_H/3$ , corresponding to a redshift of  $z_{acc} \approx 0$ . In fact, the supernova data published by the High- $z$  Supernova Search Team and the Supernova Cosmology Project show that the transition from the decelerating to the accelerating phase occurred at redshifts smaller than  $z = 0.4$  [20]. Therefore, the model predicts that the accelerated expansion phase of the Universe started only recently.

In conclusion, we have shown that the dark energy can be modeled as a massive cosmological vector field filling the Universe. This field may have originated during the inflationary period, when the vector field may have acquired its mass, and it drives the late-time acceleration of the Universe. All the parameters of the model can be constrained from observational data.

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- [1] R. R. Caldwell, Phys. World **17**, 37 (2004); T. Padmanabhan, Phys. Rep. **380**, 235 (2003); P. J. E. Peebles and P. Ratra, Rev. Mod. Phys **75**, 559 (2003).
  - [2] U. Seljak et al., Phys. Rev. D **71**, 103515 (2005).

- [3] J. A. Frieman, C. T. Hill, A. Stebbins and I. Waga, Phys. Rev. Lett. **75**, 2077 (2005); K. Coble, S. Dodelson and J. A. Frieman, Phys. Rev. D **55**, 1851 (1997); R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998); R. R. Caldwell and E. V. Linder, Phys. Rev. Lett. **95**, 141301 (2005).
- [4] C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. **85**, 4438 (2000); T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. D **62**, 023511 (2000); C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. D **63**, 103510 (2001).
- [5] A. Y. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B **511**, 265 (2001); N. Bilic, G. B. Tupper and R. D. Viollier, Phys. Lett. B **535**, 17 (2002); M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D **66**, 043507 (2002); M. K. Mak and T. Harko, Phys. Rev. D **71**, 104022 (2005).
- [6] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208 (2000); C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65**, 044023 (2002); S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Phys. Rev. D **70**, 043528 (2004).
- [7] T. Padmanabhan, Phys. Rev. D **66**, 021301 (2002); J. S. Bagla, H. K. Jassal and T. Padmanabhan, Phys. Rev. D **67**, 063504 (2003).
- [8] M. R. Garoussi, M. Sami and S. Tsujikawa, Phys. Rev. D **70**, 043536 (2004); M. R. Garoussi, M. Sami and S. Tsujikawa, Phys. Lett. B **606**, 1 (2005); M. R. Garoussi, M. Sami and S. Tsujikawa, Phys. Rev. D **71**, 083005 (2005).
- [9] V. V. Kiselev, Class. Quantum Grav. **21**, 3323 (2004); C. Armendariz-Picon, JCAP **0407**, 007 (2004).
- [10] C. M. Will and K. Nordtvedt, Jr., Astrophys. J. **177**, 757 (1972); K. Nordvedt, Jr. and C. M. Will, Astrophys. J. **177**, 775 (1972); R. W. Hellings and K. Nordtvedt, Jr., Phys. Rev. D **7**, 3593 (1973).
- [11] L. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, Butterworth Heinemann, Oxford (1998).
- [12] C. M. Will, *Theory and experiment in gravitational physics*, Cambridge University Press, Cambridge (1993).
- [13] B. Bertotti, L. Iess and P. Tortora, Nature **425**, 374 (2003).
- [14] J. G. Williams et al., Phys. Rev. D **53**, 6730 (1996).
- [15] J.-P. Uzan, Rev. Mod. Phys. **75**, 403 (2003).

- [16] J. Luo, L.-C. Tu, Z.-K. Hu, and E.-J. Luan, Phys. Rev. Lett. **90**, 081801 (2003).
- [17] C. G. Böhrmer and T. Harko, Phys. Lett. B **630**, 73 (2005).
- [18] M. S. Turner and L. M. Widrow, Phys. Rev. D **37**, 2743 (1988).
- [19] O. Bertolami and D. F. Mota, Phys. Lett. B **455**, 96 (1999); A. Ashoorioon and R. B. Mann, Phys. Rev. D **71**, 103509 (2005).
- [20] M. Turner and A. G. Riess, Astrophys. J. **569**, 18 (2002).